New agegraphic dark energy in Hořava-Lifshitz cosmology

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We investigate the new agegraphic dark energy scenario in a universe governed by Hořava-Lifshitz gravity. We consider both the detailed and non-detailed balanced version of the theory, we impose an arbitrary curvature, and we allow for an interaction between the matter and dark energy sectors. Extracting the differential equation for the evolution of the dark energy density parameter and performing an expansion of the dark energy equation-of-state parameter, we calculate its present and its low-redshift value as functions of the dark energy and curvature density parameters at present, of the Hořava-Lifshitz running parameter λ, of the new agegraphic dark energy parameter n, and of the interaction coupling b. We find that $w_0 = -0.82^{+0.08}_{-0.08}$ and $w_1 = 0.08^{+0.09}_{-0.07}$. Although this analysis indicates that the scenario can be compatible with observations, it does not enlighten the discussion about the possible conceptual and theoretical problems of Hořava-Lifshitz gravity.

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I. INTRODUCTION

Many recent cosmological observations, such as SNIa [1], WMAP [2], SDSS [3] and X-ray [4] support the idea that the universe is experiencing an accelerated expansion. A first direction that could provide an explanation of this remarkable phenomenon is to introduce the concept of dark energy, with the most obvious theoretical candidate being the cosmological constant. However, at least in an effective level, the dynamical nature of dark energy can also originate from a variable cosmological "constant" [5], or from various fields, such is a canonical scalar field (quintessence) [6], a phantom field [7], or the combination of quintessence and phantom in a unified model named quintom [8]. The second direction that could explain the acceleration is to modify the gravitational theory itself, such in the generalization to $f(R)$-gravity [9], to scalar-tensor theories with non-minimal coupling [10] etc.

Going beyond the aforementioned effective description requires a deeper understanding of the underlying theory of quantum gravity, unknown at present. However, physicists can still make some attempts to probe the nature of dark energy according to some basic quantum gravitational principles. An interesting such an attempt is the so-called "Holographic Dark Energy" proposal [11, 12]. Its framework is the black hole thermodynamics and the connection (known from AdS/CFT correspondence) of the UV cut-of of a quantum field theory, which gives rise to the vacuum energy, with the largest distance of the theory [13]. Thus, determining an appropriate quantity L to serve as an IR cut-off, imposing the constraint that the total vacuum energy in the corresponding maximum volume must not be greater than the mass of a black hole of the same size, and saturating the inequality, one identifies the acquired vacuum energy as holographic dark energy: $\rho_\Lambda = \frac{3c^2}{8\pi G L^2}$, where G is the gravitational Newton's constant and c a constant. The holographic dark energy scenario has been tested and constrained by various astronomical observations [14] and it has been extended to various frameworks [15–17].

A specific application of holographic dark energy is obtained when the age of the universe $T = \int dt$ is used as the IR cut-off L, the so-called agegraphic dark energy scenario [18]. However, since this scenario cannot describe consistently the matter-dominated period, it was extended to the new agegraphic dark energy, namely under the use of the conformal time η as the IR cut-off L [19, 20].

On the other hand, concerning the gravitational background of the universe, almost one year ago Hořava proposed a power-counting renormalizable theory with consistent ultra-violet (UV) behavior [21]. Although presenting an infrared (IR) fixed point, namely General Relativity, in the

UV the theory exhibits an anisotropic, Lifshitz scaling between time and space. Hořava-Lifshitz gravity has been studied and extended in detail [22] and it has been applied as the cosmological framework of the universe [23–26].

In the present work we are interested in investigating the new agegraphic dark energy scenario in a universe governed by Hořava-Lifshitz gravity. The plan of the paper is the following: In section II we present Hořava-Lifshitz cosmology and in section III we analyze the new agegraphic dark energy scenario. In section IV we construct the scenario of new agegraphic dark energy in Hořava-Lifshitz cosmology, both in the simple as well as in the interacting form, extracting the differential equations that determine the evolution of the dark energy density parameter. In section V we discuss the cosmological implications of such a scenario, and in particular we calculate the values and the bounds of the dark-energy equation-of-state parameter assuming a linear low-redshift parametrization. Finally, in section VI we summarize the obtained results.

II. HOŘAVA-LIFSHITZ COSMOLOGY

In this section we briefly review the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity [23]. The dynamical variables are the lapse and shift functions, N and N_i respectively, and the spatial metric g_{ij} (roman letters indicate spatial indices). In terms of these fields the full metric is written as:

$$ds^2 = -N^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{1}$$

where indices are raised and lowered using g_{ij}. The scaling transformation of the coordinates reads: $t \to l^3 t \quad \text{and} \quad x^i \to l x^i$.

A. Detailed Balance

The gravitational action is decomposed into a kinetic and a potential part as $S_g = \int dt d^3x \sqrt{g} N (\mathcal{L}_K + \mathcal{L}_V)$. The assumption of detailed balance [21] reduces the possible terms in the Lagrangian, and it allows for a quantum inheritance principle, since the $(D+1)$-dimensional theory acquires the renormalization properties of the D-dimensional one. Under the detailed balance condition the full action of Hořava-Lifshitz gravity is given by

$$\begin{aligned} S_g = \int dt d^3x \sqrt{g} N \Bigg\{ & \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2}{2w^4} C_{ij}C^{ij} - \frac{\kappa^2\mu}{2w^2}\frac{\epsilon^{ijk}}{\sqrt{g}} R_{il}\nabla_j R^l_k + \frac{\kappa^2\mu^2}{8} R_{ij}R^{ij} \\ & + \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left[\frac{1-4\lambda}{4}R^2 + \Lambda R - 3\Lambda^2\right]\Bigg\}, \end{aligned} \tag{2}$$

where

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \tag{3}$$

is the extrinsic curvature and

$$C^{ij} \; = \; \frac{\epsilon^{ijk}}{\sqrt{g}}\nabla_k\bigl(R^j_i - \frac{1}{4}R\delta^j_i\bigr) \tag{4}$$

the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric g_{ij}. ϵ^{ijk} is the totally antisymmetric unit tensor, λ is a dimensionless constant and the variables κ, w and μ are constants with mass dimensions -1, 0 and 1, respectively. Finally, we mention that in action (2) we have already performed the usual analytic continuation of the parameters μ and w of the original version of Hořava-Lifshitz gravity, since such a procedure is required in order to obtain a realistic cosmology [24] (although it could fatally affect the gravitational theory itself). Therefore, in the present work Λ is a positive constant, which as usual is related to the cosmological constant in the IR limit.

Lastly, in order to incorporate the (dark plus baryonic) matter component one adds a cosmological stress-energy tensor to the gravitational field equations, by demanding to recover the usual general relativity formulation in the low-energy limit [25]. Thus, this matter-tensor is a hydrodynamical approximation with its energy density ρ_M and pressure p_M (or ρ_M and its equation-of-state parameter $w_M \equiv p_M/\rho_M$) as parameters.

Now, in order to focus on cosmological frameworks, we impose the so called projectability condition [23] and use a Friedmann-Robertson-Walker (FRW) metric,

$$N = 1\ , \quad g_{ij} = a^2(t)\gamma_{ij}\ , \quad N^i = 0\ , \tag{5}$$

with

$$\gamma_{ij}dx^i dx^j = \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\ , \tag{6}$$

where $k = -1, 0, +1$ corresponding to open, flat, and closed universe respectively. By varying N and g_{ij}, we obtain the equations of motion:

$$H^2 = \frac{\kappa^2}{6(3\Lambda-1)}\rho_M + \frac{\kappa^2}{6(3\Lambda-1)}\left[\frac{3\kappa^2\mu^2 k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right] - \frac{\kappa^4\mu^2\Lambda k}{8(3\lambda-1)^2 a^2} \tag{7}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\Lambda-1)}w_M\rho_M - \frac{\kappa^2}{4(3\Lambda-1)}\left[\frac{\kappa^2\mu^2 k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right] - \frac{\kappa^4\mu^2\Lambda k}{16(3\lambda-1)^2 a^2}, \tag{8}$$

where we have defined the Hubble parameter as $H \equiv \frac{\dot{a}}{a}$. As usual, ρ_M follows the standard evolution equation

$$\dot{\rho}_M + 3H(1 + w_M)\rho_M = 0. \tag{9}$$

Observing the above Friedmann equations, concerning the dark-energy sector we can define

$$\rho_{DE} \equiv \frac{3\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)} \tag{10}$$

$$p_{DE} \equiv \frac{\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)}. \tag{11}$$

The term proportional to a^{-4} is the usual "dark radiation term", present in Hořava-Lifshitz cosmology [23], while the constant term is just the explicit cosmological constant. Therefore, in expressions (10),(11) we have defined the energy density and pressure for the effective dark energy, which incorporates the aforementioned contributions. Finally, note that using (10),(11) it is straightforward to show that these dark energy quantities satisfy the standard evolution equation:

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0. \tag{12}$$

As a last step, requiring these expressions to coincide with the standard Friedmann equations, in units where $c = 1$ we set [23]:

$$G_{\text{cosmo}} = \frac{\kappa^2}{16\pi(3\lambda - 1)} \tag{13}$$

$$\frac{\kappa^4\mu^2\Lambda}{8(3\lambda - 1)^2} = 1, \tag{14}$$

where G_{cosmo} is the "cosmological" Newton's constant. We mention that in theories with Lorentz invariance breaking (such is Hořava-Lifshitz one) the "gravitational" Newton's constant G_{grav}, that is the one that is present in the gravitational action, does not coincide with the "cosmological" Newton's constant G_{cosmo}, that is the one that is present in Friedmann equations, unless Lorentz invariance is restored [27]. For completeness we mention that in our case

$$G_{\text{grav}} = \frac{\kappa^2}{32\pi}, \tag{15}$$

as it can be straightforwardly read from the action (2). Thus, it becomes obvious that in the IR ($\lambda = 1$), where Lorentz invariance is restored, G_{cosmo} and G_{grav} coincide.

Using the above identifications, we can re-write the Friedmann equations (7),(8) as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{\text{cosmo}}}{3}\left(\rho_M + \rho_{DE}\right) \tag{16}$$

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_{\text{cosmo}}\left(w_M\rho_M + w_{DE}\rho_{DE}\right), \tag{17}$$

where we have introduced the effective dark energy equation-of-state parameter $w_{DE} \equiv p_{DE}/\rho_{DE}$.

B. Beyond Detailed Balance

The above formulation of Hořava-Lifshitz cosmology has been performed under the imposition of the detailed-balance condition. However, in the literature there is a discussion whether this condition leads to reliable results or if it is able to reveal the full information of Hořava-Lifshitz gravity [23]. Thus, one should study also the Friedmann equations in the case where detailed balance is relaxed. In such a case one can in general write [25]:

$$H^2 = \frac{2\sigma_0}{(3\Lambda-1)}\rho_M + \frac{2}{(3\Lambda-1)}\left[\frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6}\right] + \frac{\sigma_2}{3(3\Lambda-1)}\frac{K}{a^2} \tag{18}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{3\sigma_0}{(3\Lambda-1)}w_M\rho_M - \frac{3}{(3\Lambda-1)}\left[-\frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6}\right] + \frac{\sigma_2}{6(3\Lambda-1)}\frac{K}{a^2}, \tag{19}$$

where $\sigma_0 \equiv \kappa^2/12$, and the constants σ_i are arbitrary (with σ_2 being negative and σ_4 positive). Note that one could absorb the factor of 6 in redefined parameters, but we prefer to keep it in order to coincide with the notation of [25]. As we observe, the effect of the detailed-balance relaxation is the decoupling of the coefficients, together with the appearance of a term proportional to a^{-6}. In this case the corresponding quantities for dark energy are generalized to

$$\rho_{DE}|_{\text{non-db}} \equiv \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \tag{20}$$

$$p_{DE}|_{\text{non-db}} \equiv -\frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6}. \tag{21}$$

Again, it is easy to show that

$$\dot{\rho}_{DE}|_{\text{non-db}} + 3H(\rho_{DE}|_{\text{non-db}} + p_{DE}|_{\text{non-db}}) = 0. \tag{22}$$

Finally, if we force (18),(19) to coincide with the standard Friedmann equations, we result to:

$$G_{\text{cosmo}} = \frac{6\sigma_0}{8\pi(3\lambda-1)} \tag{23}$$

$$\sigma_2 = -3(3\lambda-1), \tag{24}$$

while in this case the "gravitational" Newton's constant G_{grav} reads [25]:

$$G_{\text{grav}} = \frac{6\sigma_0}{16\pi}. \tag{25}$$

Thus, the Friedmann equations take the standard form (16),(17) too, but with ρ_{DE} and p_{DE} given by (20),(21) respectively.

III. NEW AGEGRAPHIC DARK ENERGY

In this section we present the scenario of new agegraphic dark energy. In order to be complete, we first construct the basic model, and then we extend it in the case where the matter and dark energy sectors interact with each other. Throughout the work, we consider the background geometry to be Friedmann-Robertson-Walker.

A. The basic scenario

According to new agegraphic dark energy model [19] the dark energy sector of the universe is attributed to a holographic dark energy, where the IR cut-of of the theory is taken to be the conformal time η of the FRW universe:

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}. \tag{26}$$

Thus, the corresponding dark energy density reads:

$$\rho_{DE} = \frac{3n^2}{8\pi G_{\rm grav}\eta^2}, \tag{27}$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density is derived for Minkowski spacetime), and so on [19].

We stress here that, strictly speaking, the Newton's constant that is present in (27) is the gravitational one, since it arises form the gravitational, black-hole properties of the theory. As we discussed in the previous section, in conventional theories this gravitational Newton's constant $G_{\rm grav}$ coincides with the cosmological one $G_{\rm cosmo}$, and their distinction is not needed to be mentioned. However, since in the present work we are interested in applying new agegraphic dark energy in the framework of Hořava-Lifshitz gravity, in which $G_{\rm grav}$ and $G_{\rm cosmo}$ do not coincide unless the IR limit is reached, we prefer to maintain the distinction between $G_{\rm grav}$ and $G_{\rm cosmo}$ in order to be transparent.

As usual, in the new agegraphic dark energy scenario, the energy densities for matter and dark energy obey the standard evolution equations:

$$\dot\rho_M + 3H(\rho_M + p_M) = 0 \tag{28}$$

$$\dot\rho_{DE} + 3H(\rho_{DE} + p_{DE}) = 0. \tag{29}$$

In the following it proves more convenient to introduce the density parameters

$$\Omega_M = \frac{8\pi G_{\rm cosmo}}{3H^2}\rho_M, \quad \Omega_{DE} = \frac{8\pi G_{\rm cosmo}}{3H^2}\rho_{DE}, \quad \Omega_k = -\frac{k}{a^2H^2}. \tag{30}$$

Thus, from (30) and (27) we obtain

$$\Omega_{DE} = \left(\frac{G_{\rm cosmo}}{G_{\rm grav}}\right)\frac{n^2}{H^2\eta^2}. \tag{31}$$

Now, denoting by dot the time-derivative and by prime the derivative with respect to $\ln a$, for every quantity F we acquire $\dot{F} = F'H$. Thus, differentiating (31) and using that $\dot{\eta} = 1/a$ one obtains

$$\Omega'_{DE} = -2\Omega_{DE}\left[\frac{\dot{H}}{H^2} + \frac{\sqrt{\Omega_{DE}}}{na}\sqrt{\frac{G_{\rm grav}}{G_{\rm cosmo}}}\right]. \tag{32}$$

Finally, differentiating (27) we obtain

$$\dot{\rho}_{DE} = -\frac{2\rho_{DE}}{a\eta}, \tag{33}$$

which allows us to use (29) in order to define the new agegraphic dark energy equation-of-state parameter as [19, 20]:

$$w_{DE} = -1 + \frac{2}{3na}\sqrt{\Omega_{DE}}\sqrt{\frac{G_{\rm grav}}{G_{\rm cosmo}}}. \tag{34}$$

B. The interacting scenario

A valuable extension of the aforementioned basic model, is the one in which the matter and dark energy sectors are allowed to interact [28], since such a scenario could alleviate the known coincidence problem [29]. In such a case the evolution equations for the matter and dark energy densities write:

$$\dot{\rho}_M + 3H(\rho_M + p_M) = Q \tag{35}$$

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q. \tag{36}$$

A usual and quite general ansatz for the interaction term Q is $Q = 3bH(\rho_M + \rho_{DE})$ [19], with b a coupling parameter (we prefer to call the coupling b and not b^2 in order to coincide with the majority of the authors, since a negative value could be possible too [30, 31]). Therefore, $b < 0$ corresponds to energy transfer from dark matter to dark energy, while $b > 0$ corresponds to dark energy transformation to dark matter. Finally, in this case, relations (31), (32) and (34) are valid too.

IV. NEW AGEGRAPHIC DARK ENERGY IN HOŘAVA-LIFSHITZ GRAVITY

Let us now construct the scenario in which new agegraphic dark energy is applied in a universe governed by Hořava-Lifshitz gravity. That is, we will insert the relations (31) and (32) of section III in the Friedmann equations derived of section II. In order to be complete, we perform this separately for the basic and for the interacting case. Finally, note that we perform our analysis for a general curvature, since, as it has been extensively stated in the literature [23], Hořava-Lifshitz cosmology coincides completely with ΛCDM if one ignores curvature.

A. The basic scenario

Let us use the first Friedmann equation (16) of Hořava-Lifshitz cosmology, in order to eliminate the term $\dot{H}/H^2$ that is present in new agegraphic dark energy evolution (32). In order to simplify the presentation we consider as usual that the matter is dust, that is $w_M = 0$. Differentiating (16), using (33) and (28), and inserting the density parameters (30), we obtain

$$\frac{\dot{H}}{H^2} = -\Omega_k - \frac{3}{2}\Omega_M - \frac{(\Omega_{DE})^{3/2}}{an}\sqrt{\frac{G_{\text{grav}}}{G_{\text{cosmo}}}}. \tag{37}$$

Therefore, inserting this expression into (32), and using that $\Omega_M = 1 - \Omega_{DE} - \Omega_k$, we finally acquire:

$$\Omega'_{DE} = \Omega_{DE}\left[3(1-\Omega_{DE}) - \Omega_k + 2\sqrt{\frac{G_{\text{grav}}}{G_{\text{cosmo}}}}\frac{\sqrt{\Omega_{DE}}}{an}(\Omega_{DE}-1)\right]. \tag{38}$$

A final step is to use the definitions of G_{grav} and G_{cosmo} in Hořava-Lifshitz cosmology. According to (13) and (15) the ratio $G_{\text{grav}}/G_{\text{cosmo}}$ is equal to $(3\lambda-1)/2$ in the detailed balance version of the theory, and it takes the same value in the beyond detail-balance version too, as it can be extracted from (23) and (25). Therefore, in both versions of Hořava-Lifshitz gravity, the differential equation that determines the evolution of the new agegraphic dark energy reads:

$$\Omega'_{DE} = \Omega_{DE}\left[3(1-\Omega_{DE}) - \Omega_k + 2\sqrt{\frac{3\lambda-1}{2}}\frac{\sqrt{\Omega_{DE}}}{an}(\Omega_{DE}-1)\right]. \tag{39}$$

B. The interacting scenario

Let us repeat the above procedure in the case where the matter and dark energy sectors are allowed to interact. Thus, differentiating (16), using (33) and (35), we obtain:

$$\frac{\dot{H}}{H^2} = -\Omega_k - \frac{3}{2}\Omega_M - \frac{(\Omega_{DE})^{3/2}}{an}\sqrt{\frac{G_{\text{grav}}}{G_{\text{cosmo}}}} + \frac{3}{2}b(\Omega_M + \Omega_{DE}). \tag{40}$$

Therefore, inserting this relation into (32), and using also that $G_{\rm grav}/G_{\rm cosmo} = (3\lambda-1)/2$, we acquire:

$$\Omega'_{DE} = \Omega_{DE}\left[3(1-\Omega_{DE}) - \Omega_k + 2\sqrt{\frac{3\lambda-1}{2}}\frac{\sqrt{\Omega_{DE}}}{an}(\Omega_{DE}-1) - b(1-\Omega_k)\right]. \tag{41}$$

Finally, note that this relation holds in general, either in the detailed-balance case, or beyond it.

V. COSMOLOGICAL IMPLICATIONS

In the above sections we have formulated the new agegraphic dark energy model in a universe governed by Hořava-Lifshitz gravity, and in particular we extracted the differential equation that determines the evolution of the dark energy density parameter Ω_{DE}. Thus, in the present section we use this expression in order to calculate the basic observable, namely the dark energy equation-of-state parameter w_{DE} at present as well as at small redshifts. In order to achieve this we perform the standard expansions of the literature. In particular, since $\rho_{DE} \sim a^{-3(1+w_{DE})}$ we acquire [12]

$$\ln\rho_{DE} = \ln\rho_{DE0} + \frac{d\ln\rho_{DE}}{d\ln a}\ln a + \frac{1}{2}\frac{d^2\ln\rho_{DE}}{d(\ln a)^2}(\ln a)^2 + \dots\ , \tag{42}$$

where the derivatives are calculated at the present time $a_0 = 1$ and the index 0 marks the value of a quantity at present. Then, $w_{DE}(\ln a)$ is given as

$$w_{DE}(\ln a) = -1 - \frac{1}{3}\left[\frac{d\ln\rho_{DE}}{d\ln a} + \frac{1}{2}\frac{d^2\ln\rho_{DE}}{d(\ln a)^2}\ln a\right], \tag{43}$$

up to second order. Since

$$\rho_{DE} = \frac{3H^2\Omega_{DE}}{8\pi G_{\rm grav}} = \left(\frac{G_{\rm cosmo}}{G_{\rm grav}}\right)\frac{\rho_M}{\Omega_M}\Omega_{DE} = \left(\frac{2}{3\lambda-1}\right)\frac{\rho_{m0}a^{-3}\Omega_{DE}}{(1-\Omega_k-\Omega_{DE})}, \tag{44}$$

(which holds for dust matter) the derivatives in the w_{DE}-expansion are easily computed using the obtained expressions for Ω'_{DE}. In addition, we can straightforwardly calculate $w_{DE}(z)$, that is using the redshift z as the independent variable, replacing $\ln a = -\ln(1+z) \simeq -z$, which is valid for small redshifts. Doing so we obtain:

$$w_{DE}(z) = -1 - \frac{1}{3}\left(\frac{d\ln\rho_{DE}}{d\ln a}\right) + \frac{1}{6}\left[\frac{d^2\ln\rho_{DE}}{d(\ln a)^2}\right]z \equiv w_0 + w_1 z. \tag{45}$$

A. The basic scenario

In this case Ω'_{DE} is given by (39) and the aforementioned differentiation procedure leads to:

$$w_0 = \frac{(1-\Omega_{k0})}{3n(1-\Omega_{k0}-\Omega_{DE0})}\left\{n(\Omega_{k0}-3)+\sqrt{2(3\lambda-1)}\sqrt{\Omega_{DE0}}(1-\Omega_{DE0})+3n\Omega_{DE0}\right\} \qquad (46)$$

$$\begin{aligned}
w_1 = &\frac{(\Omega_{k0}-1)\Omega_{DE0}}{12n\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}(-1+\Omega_{k0}+\Omega_{DE0})^2}\left\{\sqrt{2}n(3\lambda-1)(\Omega_{k0}-1)^2\right.\\
&+2n^2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}\left[2(\Omega_{k0}-3)\Omega_{k0}\right]\\
&+2(\Omega_{k0}-1)(3\lambda-1)^{3/2}\sqrt{\Omega_{DE0}}+(3\lambda-1)\Omega_{DE0}^3\left(2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}-3\sqrt{2}n\right)\\
&+(3\lambda-1)\Omega_{DE0}^2\left\{6(\Omega_{k0}-1)\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}-\sqrt{2}n\left[14\Omega_{k0}-7\right]\right\}\\
&+\Omega_{DE0}\left\{\sqrt{2}n(3\lambda-1)\left\{\Omega_{k0}\left[16-3\Omega_{k0}\right]-5\right\}\right.\\
&\qquad\left.\left.+12n^2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}\Omega_{k0}-2(3\lambda-1)^{3/2}\sqrt{\Omega_{DE0}}(4\Omega_{k0}-3)\right\}\right\}. \qquad (47)
\end{aligned}$$

These expressions provide w_0 and w_1, for the basic scenario. Despite their complicated form they can be very helpful since they involve only the present density parameters Ω_{k0},Ω_{DE0}, the running parameter λ of Hořava-Lifshitz gravity, and the parameter n of new agegraphic dark energy. Ω_{k0},Ω_{DE0} are known in a good accuracy, namely $\Omega_{DE0} = 0.728^{+0.015}_{-0.016}$ and $\Omega_{k0} = -0.013^{+0.006}_{-0.007}$ in 1σ [32]. Additionally, observational constraints restrict λ in a narrow window around its IR value 1 namely $|\lambda-1| \leq 0.02$ in 1σ [33]. Thus, the larger uncertainty comes from the value of the parameter n, which varies in the bound $n = 2.72^{+0.11}_{-0.11}$ in 1σ [34]. Inserting these values and variation intervals into (46),(47) we obtain that in 1σ:

$$\begin{aligned}
w_0 &= -0.780^{+0.022}_{-0.022}\\
w_1 &= 0.050^{+0.019}_{-0.018}.
\end{aligned} \qquad (48)$$

As we observe, the value of the present dark-energy equation-of-state parameter w_0 is slightly larger than the standard new agegraphic dark energy model [34], however, as expected, its 1σ-bounds are significantly larger due to the additional uncertainties in the values of n and λ. Finally, note that according to these values the scenario at hand cannot exhibit the phantom-divide crossing. This is a feature of the conventional new agegraphic dark energy model [19, 20, 34] and it is inherited by the Hořava-Lifshitz one too, since the increased bounds due to the additional uncertainties are not too wide in order to cover the values below -1.

B. The interacting scenario

For the interacting scenario, Ω'_{DE} is given by (41) and repeating the above steps we result to:

$$w_0 = \frac{(1-\Omega_{k0})}{3n(1-\Omega_{k0}-\Omega_{DE0})}\left\{n\left[\Omega_{k0}-3+b(\Omega_{k0}-1)\right]+\sqrt{2(3\lambda-1)}\sqrt{\Omega_{DE0}}(1-\Omega_{DE0})+3n\Omega_{DE0}\right\} \tag{49}$$

$$\begin{aligned}
w_1 = & \frac{(\Omega_{k0}-1)\Omega_{DE0}}{12n\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}(-1+\Omega_{k0}+\Omega_{DE0})^2}\Big\{-\sqrt{2}n(3\lambda-1)(b-1)(\Omega_{k0}-1)^2 \\
& -2n^2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}\left[b^2(\Omega_{k0}-1)^2-2(\Omega_{k0}-3)\Omega_{k0}+b(\Omega_{k0}-1)(3+\Omega_{k0})\right] \\
& +2(\Omega_{k0}-1)(3\lambda-1)^{3/2}\sqrt{\Omega_{DE0}}+(3\lambda-1)\Omega_{DE0}^3\left(2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}-3\sqrt{2}n\right) \\
& +(3\lambda-1)\Omega_{DE0}^2\left\{6(\Omega_{k0}-1)\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}-\sqrt{2}n\left[b(\Omega_{k0}-1)+14\Omega_{k0}-7\right]\right\} \\
& +\Omega_{DE0}\Big\{\sqrt{2}n(3\lambda-1)\left\{\Omega_{k0}\left[3b(\Omega_{k0}-1)-3\Omega_{k0}+16\right]-5\right\} \\
& \quad +6n^2\sqrt{3\lambda-1}\sqrt{\Omega_{DE0}}\left[b(\Omega_{k0}-1)+2\Omega_{k0}\right]-2(3\lambda-1)^{3/2}\sqrt{\Omega_{DE0}}(4\Omega_{k0}-3)\Big\}\Big\}.
\end{aligned} \tag{50}$$

Similarly to the previous subsection, we can insert the values and the 1σ-variation bounds for the model parameters Ω_{k0},Ω_{DE0}, λ, n and the coupling parameter b, in order to extract the corresponding values for w_0 and w_1. For the first four we use $\Omega_{DE0} = 0.728^{+0.015}_{-0.016}$, $\Omega_{k0} = -0.013^{+0.006}_{-0.007}$ [32], $|\lambda - 1| \leq 0.02$ [33] and $n = 2.72^{+0.11}_{-0.11}$ [34]. For the coupling parameter b there are various observational constraints in the literature [30, 31, 35], all of which lie in a narrow window around zero, thus we will use a representative interval $-0.08 < b < 0.03$ [30]. Inserting these values and variation intervals into (49),(50), within 1σ we obtain

$$\begin{aligned}
w_0 &= -0.82^{+0.08}_{-0.08} \\
w_1 &= 0.08^{+0.09}_{-0.07}.
\end{aligned} \tag{51}$$

As we observe, the value of w_0 is smaller than the corresponding one of the non-interacting scenario above, while w_1 is significantly larger. Furthermore, the bounds of both these parameters are significantly larger, due to the additional uncertainty in the coupling b. Finally, note moreover that these bounds are also larger from the corresponding ones in interacting new agegraphic dark energy models in conventional cosmology [34], due to the uncertainty in the running parameter λ of Hořava-Lifshitz gravity. However, they are still not so large in order to make the phantom-divide crossing possible.

VI. CONCLUSIONS

In this work we investigated the new agegraphic dark energy scenario in a universe governed by Hořava-Lifshitz gravity. In order to be general we considered both versions of the theory,

that is with or without the detailed-balance condition, we imposed an arbitrary curvature for the background geometry, and we allowed for an interaction between the matter and dark energy sectors. In both the basic and interacting case we extracted the differential equation that determines the evolution of the dark energy density parameter, which is independent of the detailed-balance condition. Finally, using this equation and performing a low-redshift expansion of the dark energy equation-of-state parameter $w(z) \approx w_0 + w_1 z$, we calculated w_0 and w_1 as functions of the dark energy and curvature density parameters at present, Ω_{DE0} and Ω_{k0} respectively, of the running parameter λ of Hořava-Lifshitz gravity, of the parameter n of new agegraphic dark energy, and of the interaction coupling b.

In the non-interacting scenario, we found that $w_0 = -0.780^{+0.022}_{-0.022}$ is slightly larger comparing to the standard new agegraphic dark energy model [34], however its 1σ-bounds are significantly larger, as expected, due to the additional uncertainties in the values of n and λ. Furthermore, in the interacting case, $w_0 = -0.82^{+0.08}_{-0.08}$ is smaller than the corresponding one of the non-interacting model, while $w_1 = 0.08^{+0.09}_{-0.07}$ is significantly larger. Moreover, the bounds of both w_0 and w_1 are significantly larger, due to the additional uncertainty in the interaction parameter b. Finally, note that these bounds are also larger from the corresponding ones in interacting new agegraphic dark energy models in standard cosmology [34], due to the uncertainty in the running parameter λ of Hořava-Lifshitz gravity. However, in both the basic and interacting scenarios, the increased bounds in w_0 are still not too large in order to make the phantom-divide crossing possible.

It is interesting to note that the scenario of new agegraphic dark energy in Hořava-Lifshitz cosmology seems to be more efficient than that of holographic dark energy in the same gravitational framework in a flat universe [36]. This feature acts as an advantage of the present scenario, and indicates that if the underlying gravitational theory is indeed the Hořava-Lifshitz one and if dark energy exhibits a holographic nature, then the new agegraphic version in a non-flat geometry should be used instead of the simple (event-horizon) holographic one.

We close this work by mentioning that although the present analysis indicates that new agegraphic dark energy in Hořava-Lifshitz cosmology can be consistent and compatible with observations, it does not enlighten the discussion about possible conceptual problems and instabilities of Hořava-Lifshitz gravity, nor it can interfere with the questions concerning the validity of its theoretical background, which is the subject of interest of other studies. It just faces the dark energy problem in such a context, and thus its results can be taken into account only if Hořava-Lifshitz gravity passes successfully the necessary theoretical tests.

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